

# The Standard Model instability and the scale of new physics <sup>\*</sup>

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## Abstract

We apply a general formalism for the improved effective potential with several mass scales to compute the scale  $M$  of new physics which is needed to stabilize the Standard Model potential in the presence of a light Higgs. We find, by imposing perturbativity of the new physics, that  $M$  can be as large as one order of magnitude higher than the instability scale of the Standard Model. This implies that, with the present lower bounds on the Higgs mass, the new physics could easily (but not necessarily) escape detection in the present and future accelerators.

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The Standard Model (SM) effective potential is unstable beyond a scale,  $\Lambda_{SM}$ , which solely depends upon the value of the (yet undiscovered) Higgs mass  $M_H$  [1]<sup>1</sup>. This instability is a drawback of the Standard Model, which is unable to describe physics at scales beyond  $\Lambda_{SM}$  and, then, requires the presence of new physics to stabilize the SM vacuum. For large values of the Higgs mass the scale  $\Lambda_{SM}$  is larger than the Planck scale and thus has no impact on the physics at present and future accelerators. However, for the case of a light Higgs (i.e.  $M_H \lesssim 140$  GeV)  $\Lambda_{SM}$  can be closer to values that could directly, or indirectly, be detected at future accelerators, thus having an impact on the physics at the corresponding scales. In particular, and to guarantee the absolute stability of the electroweak vacuum, new physics must be introduced. The direct, or indirect, detection of the new physics depends on the mass,  $M$ , of the new involved fields, as deduced from the stability requirement. This is the issue that will be considered in this paper.

Since the presence of extra bosonic (fermionic) degrees of freedom tend to stabilize (destabilize) the SM Higgs potential [3, 4, 5, 6], it is clear that the simplest and most economic SM extension that could circumvent the instability problem is the addition of just scalar fields coupled to the SM Higgs field [3, 4]. This will be the model considered in the present paper. A similar analysis of this kind of model was performed by Hung and Sher in Ref. [3], where the effective potential was considered in the one-loop approximation. However improving the effective potential by the renormalization group equations (RGE) yields very important corrections and must be taken into account in any realistic calculation. Obviously, other SM extensions, such as the MSSM, can be much more realistic, but, for the previous reasons, the simple model under consideration basically represents the most efficient way to cure the SM instability, thus giving reliable upper bounds on the scale of new physics, i.e. on the mass of the extra particles.

Improving the effective potential in a SM extension generally implies considering a multi-scale problem. In our case, we have to consider two different mass scales: the SM scale, say  $\mu_t$ , that can be identified with the top-quark mass<sup>2</sup>, and the scale of new physics, say  $\mu_\Phi$ , that should be identified with the mass of the new scalar fields ( $\Phi$ ). A general formalism to evaluate the effective potential in this kind of multi-scale scenario was presented by the authors in Ref. [7], where the general procedure for decoupling

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<sup>1</sup>The original studies considered  $\Lambda_{SM}$  as a function of  $M_H$  and  $M_t$  (the top-quark mass) [2]. In this paper we will fix  $M_t$  to its experimental mean value and disregard the effect due to the experimental error  $\Delta M_t$ .

<sup>2</sup>For the sake of simplicity we will consider  $\mu_t$  as the only SM scale, and will not make a distinction with the other (nearby) SM scales, as the masses of the Higgs and the gauge bosons.

The outline of the paper is as follows: In section 2 we will briefly summarize our general proposal of multi-scale effective potential. In section 3 we will apply the ideas and results of section 2 to the model we are considering to remove the SM instability. In section 4 we will present our numerical results and in section 5 our conclusions.

## 2 Multi-scale effective theory

We will consider the effective potential of a theory with  $N$  different scales in a mass-independent renormalization scheme, as e.g. the  $\overline{\text{MS}}$  scheme, where the decoupling is not automatically incorporated in the theory. The usual procedure is to decouple every field at a given scale,  $\mu_i$ , that is normally associated to its mass, by means of e.g. a step function. However, the scale invariance of the complete effective potential indicates that the results should not depend on the details of the chosen decoupling scale  $\mu_i$ , thus implying independence of the (complete) effective action with respect to the scale  $\mu_i$  (similar to the usual scale invariance). This will give rise to a set of RGE with respect to the scales  $\mu_i$ , as well as with respect to  $\mu$ , which is the ordinary  $\overline{\text{MS}}$  renormalization scale. On top of that we will use a simple step function for decoupling<sup>3</sup>, although it could be easily smoothed. These ideas were presented in Ref. [7] and similar ones can be found in Refs. [9] and [10].

In this decoupling approach, when computing the one-loop  $\beta$ -functions corresponding to  $\lambda_a$  [where  $\lambda_a$  denotes all dimensionless and dimensional couplings of the theory, including the bosonic and fermionic fields], the contribution from decoupled fields should not be counted. This translates into the decomposition:

$$\mu \frac{d \lambda_a}{d \mu} \equiv {}_{\mu} \beta_{\lambda_a} = \sum_{i=1}^N {}_{\mu_i} \tilde{\beta}_{\lambda_a} \theta(\mu - \mu_i) \quad (2.1)$$

which provides the definition of the factors  ${}_{\mu_i} \tilde{\beta}_{\lambda_a}$ .

Invariance of the complete effective potential with respect to the  $\overline{\text{MS}}$  scale  $\mu$  simply reads

$$\mu \frac{d V_{\text{eff}}}{d \mu} = 0 \quad (2.2)$$

On the other hand, the one-loop effective potential can be written as:

$$V_{\text{eff}} = V_0 + V_1 \quad (2.3)$$

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<sup>3</sup>In addition, at a given threshold the corresponding symmetry of the system may change, in which case the corresponding matching conditions have to be taken into account at the corresponding thresholds. A typical example is the threshold of supersymmetric particles in the MSSM. Below it, the theory is non-supersymmetric and beyond it supersymmetric. This kind of complications will not appear, however, in the model at consideration in the present paper.

$$V_1 = \frac{1}{64\pi^2} \sum_{i=1}^N (-)^{2s_i} M_i^4 \left[ \log \frac{M_i^2}{\mu_i^2} + \theta(\mu - \mu_i) \log \frac{\mu_i^2}{\mu^2} - C_i \right] \quad (2.4)$$

where  $s_i$  is the spin of the  $i$ -th field,  $M_i$  are the (tree-level) mass eigenvalues and  $C_i$  depends on the renormalization scheme. In the  $\overline{\text{MS}}$ -scheme it is equal to  $3/2$  ( $5/6$ ) for scalar bosons and fermions (gauge bosons). Notice that for  $\mu > \mu_i$  (for all  $i$ ) the potential (2.4) coincides with the usual  $\overline{\text{MS}}$  effective potential when there are no decoupled particles. For  $\mu < \mu_i$  the term  $M_i^4 \log(M_i^2/\mu_i^2)$  can be taken (at the one-loop level) as frozen at the scale  $\mu = \mu_i$ , so it does not run with respect to  $\mu$ . On the other hand, the invariance of the effective potential with respect to  $\mu_i$ ,

$$\mu_i \frac{dV_{\text{eff}}}{d\mu_i} = 0 \quad (2.5)$$

leads to the running of the parameters with respect to the scale  $\mu_i$ :

$$\mu_i \frac{d\lambda_a}{d\mu_i} \equiv {}_{\mu_i}\beta_{\lambda_a} = {}_{\mu_i}\tilde{\beta}_{\lambda_a} \theta(\mu_i - \mu) \quad (2.6)$$

From Eqs. (2.2) and (2.6) it follows that

$${}_{\mu}\beta_{\lambda_a} + \sum_{i=1}^N {}_{\mu_i}\beta_{\lambda_a} = \sum_{i=1}^N {}_{\mu_i}\tilde{\beta}_{\lambda_a} = \beta_{\lambda_a}^{\overline{\text{MS}}} \quad (2.7)$$

where  $\beta_{\lambda_a}^{\overline{\text{MS}}}$  is the usual (complete)  $\beta$ -function in the  $\overline{\text{MS}}$ -scheme.

The invariance of the effective potential with respect to  $\mu$  and  $\mu_i$  allows to choose any values for these scales. These can be constant, as it is usually done [11], or field-dependent<sup>4</sup>. A choice that is particularly convenient to greatly improve the validity of perturbation theory is  $\mu_i \simeq M_i$  and  $\mu \lesssim \min\{\mu_i\}$  [7]. This gets rid of all the dangerous logarithms in Eq. (2.4). Notice here that since  $M_i$  are in general functions of the fields, so the preferred value of the  $\mu_i$  scales are. In addition, the evaluation of the effective potential requires to evaluate all the  $\lambda_a$  parameters at the same values of  $\mu_i$  (note that  $\lambda_a$  run with the  $\mu_i$  scales). This implies a knowledge of the  ${}_{\mu_i}\tilde{\beta}_{\lambda_a}$  functions defined in Eq. (2.1).

It is interesting to note that the previous decoupling approach can be obtained starting with the bare lagrangian written in an appropriate renormalization scheme. The latter is a generalization of the so-called multi-scale renormalization scheme proposed in Refs. [9, 10]. Given a set of bosonic  $\phi_i$  and fermionic  $\psi_j$  fields, the bare lagrangian,

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<sup>4</sup>This is similar to the ordinary  $\overline{\text{MS}}$ -scheme where the scale  $\mu$  can be fixed to a field dependent value. In the case of the SM this value is usually  $\sim M_t$ , in order to improve the validity of perturbation theory.

be written as:

$$\mathcal{L}_{\text{Bare}} = \sum_i \mathcal{L}_{\text{kin}, \phi_i} + \sum_j \mathcal{L}_{\text{kin}, \psi_j} + \mathcal{L}_{\text{Bare, int}} \quad (2.8)$$

where we have included in  $\mathcal{L}_{\text{Bare, int}}$  all interaction and potential terms. More precisely,

$$\begin{aligned} \mathcal{L}_{\text{kin}, \phi_i} &= \frac{1}{2} \tilde{\mu}_i^{-\varepsilon/2} f_{\phi_i}(\tilde{\mu}_\ell) Z_{\phi_i} (\partial_\mu \phi_i)^2 \\ \mathcal{L}_{\text{kin}, \psi_j} &= i \tilde{\mu}_j^{-\varepsilon/4} f_{\psi_j}(\tilde{\mu}_\ell) Z_{\psi_j} \bar{\psi}_j \partial_\mu \gamma^\mu \psi_j \\ \mathcal{L}_{\text{Bare, int}} &= \sum_b Z_{\lambda_b} f_{\lambda_b}(\tilde{\mu}_\ell) \lambda_b O_b(Z_{\phi_i}^{1/2} \phi_i, Z_{\psi_j}^{1/2} \psi_j) \end{aligned} \quad (2.9)$$

where  $Z_{\phi_i}$  ( $Z_{\psi_j}$ ) and  $Z_{\lambda_b}$  are the bosonic (fermionic) wave function renormalizations and the coupling renormalizations respectively, and  $O_b(\phi_i, \psi_j)$  represent interaction operators between the fields. The  $\tilde{\mu}_i$  scales are appropriate combinations of the independent scales  $\mu$  and  $\mu_i$ , namely

$$\log \tilde{\mu}_i = \log \mu \theta(\mu - \mu_i) + \log \mu_i \theta(\mu_i - \mu) \quad (2.10)$$

Finally, the functions  $f_{\phi_i}$ ,  $f_{\psi_j}$  and  $f_{\lambda_b}$  are dimensionless functions of the ratios  $\tilde{\mu}_i/\tilde{\mu}_j$  which are constant and finite and can be expanded as:  $f = 1 + \mathcal{O}(\hbar)$ . They correspond to finite wave-function and coupling renormalizations. They should be chosen so that the  $\beta$  and  $\gamma$ -functions obtained from the bare lagrangian satisfy the decomposition given by Eqs. (2.1) and (2.6).

Let us notice that for  $\mu < \mu_i$  (all  $i$ ), we have  $\tilde{\mu}_i = \mu_i$ , while for  $\mu > \mu_i$  (all  $i$ ), we have  $\tilde{\mu}_i = \mu$ . In the latter region, we recover the ordinary  $\overline{\text{MS}}$ -scheme. At intermediate values of  $\mu$ , the situation is also intermediate: some of the  $\tilde{\mu}_i$  become equal to the  $\overline{\text{MS}}$ -scale  $\mu$ .

It can be checked that the one-loop effective potential obtained from the lagrangian (2.8) has precisely the form of the proposed one-loop effective potential of Eq. (2.4). In terms of the  $\tilde{\mu}_i$  scales defined in Eq. (2.10), it simply reads

$$V_1 = \frac{1}{64\pi^2} \sum_{i=1}^N (-)^{2s_i} M_i^4 \left[ \log \frac{M_i^2}{\tilde{\mu}_i^2} - C_i \right] \quad (2.11)$$

In the next section we will apply this approach to our simple extension of the Standard Model.

### 3 Effective theory of the Standard Model extension

The presence of extra bosonic (fermionic) degrees of freedom tends to stabilize (destabilize) the SM Higgs potential. Consequently, when the SM Higgs potential presents

extra bosonic fields. Consequently, in this section we will apply the results of section 2 to a very simple extension of the Standard Model defined by the lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + \frac{1}{2} \left( \partial_\mu \vec{\Phi} \right)^2 - \frac{1}{2} \delta |H|^2 \vec{\Phi}^2 - \frac{1}{2} M^2 \vec{\Phi}^2 - \frac{1}{4!} \lambda_s \vec{\Phi}^4 \quad (3.1)$$

where  $\mathcal{L}_{\text{SM}}$  is the SM lagrangian,  $H$  the SM Higgs doublet and  $\vec{\Phi}$   $N$  real scalar fields transforming under the vector representation of  $O(N)$ .  $M$  is the invariant mass of  $\vec{\Phi}$ ,  $\lambda_s$  its quartic coupling and  $\delta$  provides the mixing between the Higgs and  $\vec{\Phi}$  fields. Since we are assuming that  $\vec{\Phi}$  is a SM singlet,  $\delta$  is the only coupling that involves the SM with the new physics and will play a relevant role in our analysis. Of course, this model may be unrealistic, but, for the previous reasons, it basically represents the most efficient way to cure a SM instability, thus giving reliable upper bounds on the scale of the required new physics, i.e. the mass of the extra particles.

In the present case we have two relevant scales: the SM scale, which can be conventionally chosen to be that corresponding to the top-quark,  $\mu_t$ , and the scale of the new physics,  $\mu_\Phi$ . In the background of the Higgs field,  $H^0 = h_c/\sqrt{2}$ , the one-loop effective potential in the decoupling approach explained in the previous section can be decomposed as in Eq. (2.3) with

$$V_0 = -\frac{1}{2} m^2 h_c^2 + \frac{1}{4} \lambda h_c^4 + \Lambda_c \quad (3.2)$$

where  $m^2$ ,  $\lambda$  and  $\Lambda_c$  are the SM mass term, quartic coupling and vacuum energy, respectively, and  $V_1$  as given by (2.4) [or, equivalently, by (2.11)]

$$\begin{aligned} V_1 = & \frac{1}{64\pi^2} \left[ -12M_t^4(h_c) \left( \log \frac{M_t^2(h_c)}{\mu_t^2} + \theta(\mu - \mu_t) \log \frac{\mu_t^2}{\mu^2} - \frac{3}{2} \right) \right. \\ & \left. + N M_\Phi^4(h_c) \left( \log \frac{M_\Phi^2(h_c)}{\mu_\Phi^2} + \theta(\mu - \mu_\Phi) \log \frac{\mu_\Phi^2}{\mu^2} - \frac{3}{2} \right) \right] . \end{aligned} \quad (3.3)$$

The masses of the top-quark and the  $\vec{\Phi}$  field are defined by

$$\begin{aligned} M_t^2(h_c) &= \frac{\lambda_t h_c^2}{\sqrt{2}} \\ M_\Phi^2(h_c) &= M^2 + \delta h_c^2 , \end{aligned} \quad (3.4)$$

$\lambda_t$  being the SM top-quark Yukawa coupling. We are neglecting in Eq. (3.3) the contribution to the one-loop effective potential of all SM fields, except the top-quark field, as it is usually done in SM studies.

To evaluate the potential given by Eqs. (3.2, 3.3) we also need to know the  $\lambda_a$  parameters (i.e. all the masses, couplings and fields) at the corresponding values of the

and (2.6) for the relevant parameters of our model. This is given by

$$\mu_t \tilde{\beta}_\lambda = \beta_\lambda^{\text{SM}} \quad \mu_\Phi \tilde{\beta}_\lambda = \frac{2N}{\kappa} \delta^2 \quad (3.5)$$

$$\mu_t \tilde{\beta}_{m^2} = \beta_{m^2}^{\text{SM}} \quad \mu_\Phi \tilde{\beta}_{m^2} = -\frac{2N}{\kappa} \delta M^2 \quad (3.6)$$

$$\mu_t \tilde{\beta}_\Lambda = \beta_\Lambda^{\text{SM}} \quad \mu_\Phi \tilde{\beta}_\Lambda = \frac{N}{2\kappa} M^4 \quad (3.7)$$

$$\mu_t \tilde{\gamma}_h = \gamma_h^{\text{SM}} \quad \mu_\Phi \tilde{\gamma}_h = 0 \quad (3.8)$$

for the SM couplings, where  $\kappa = 16\pi^2$ , and  $\beta^{\text{SM}}, \gamma^{\text{SM}}$  are the SM  $\beta$  and  $\gamma$ -functions. For couplings corresponding to new physics the  $\tilde{\beta}$ -functions are:

$$\begin{aligned} \mu_t \tilde{\beta}_\delta &= 2\gamma_h \delta + \frac{24}{\kappa} \lambda \delta \\ \mu_\Phi \tilde{\beta}_\delta &= \frac{1}{\kappa} \left( 8\delta^2 + \frac{1}{3}(N+2)\lambda_s \delta \right) \end{aligned} \quad (3.9)$$

$$\begin{aligned} \mu_t \tilde{\beta}_{\lambda_s} &= \frac{48}{\kappa} \delta^2 \\ \mu_\Phi \tilde{\beta}_{\lambda_s} &= \frac{1}{3\kappa} (N+8)\lambda_s^2 \end{aligned} \quad (3.10)$$

$$\begin{aligned} \mu_t \tilde{\beta}_{M^2} &= -\frac{8}{\kappa} \delta m^2 \\ \mu_\Phi \tilde{\beta}_{M^2} &= \frac{1}{3\kappa} \lambda_s (N+2) M^2 \end{aligned} \quad (3.11)$$

As noted in the previous section, and is apparent from (3.3), for scales  $\mu > \mu_t, \mu_\Phi$ , the effective potential (3.3) reduces to the ordinary  $\overline{\text{MS}}$  effective potential. However, for an improved evaluation of the potential it is much more convenient to choose  $\mu \simeq \mu_t \simeq M_t(h_c)$ ,  $\mu_\Phi \simeq M_\Phi(h_c)$ , thus getting rid of dangerous logarithms. (In the next section we will be more precise about the exact values of these choices.) These values belong to the range  $\mu_t \lesssim \mu < \mu_\Phi$ , where the one-loop effective potential can be written as:

$$\begin{aligned} V_1 &= V_1^{\text{SM}} + V_1^\Phi \\ V_1^\Phi &= \frac{N}{64\pi^2} M_\Phi^4(h_c) \left( \log \frac{M_\Phi^2(h_c)}{\mu_\Phi^2} - \frac{3}{2} \right) \end{aligned} \quad (3.12)$$

Here  $V_1^{\text{SM}}$  is the usual SM one loop-effective potential in the  $\overline{\text{MS}}$ -scheme, which depends explicitly on the RGE scale  $\mu$ , while  $V_1^\Phi$  corresponds to the contribution of the decoupled field  $\vec{\Phi}$ , which runs with  $\mu_\Phi$ . Expressions (3.2, 3.12) will be used in the next section to evaluate explicitly the effective potential.

To finish this section, it is interesting to write the explicit form of the bare lagrangian in a multiscale renormalization scheme [see Eqs.(2.8, 2.9)] which leads to the effective

scales,  $\mu_t \lesssim \mu < \mu_\Phi$  [which, by Eq. (2.10) implies  $\tilde{\mu}_t = \mu$ ,  $\tilde{\mu}_\Phi = \mu_\Phi$ ], this is explicitly given by

$$\begin{aligned} \mathcal{L}_{\text{Bare}} &= \mathcal{L}_{\text{Bare}}^{\text{SM}} \\ &+ \frac{1}{2} \mu_\Phi^{-\varepsilon/2} Z_\Phi \left( \partial_\mu \vec{\Phi} \right)^2 - \frac{1}{2} Z_\Phi Z_{M^2} M^2 \vec{\Phi}^2 - \frac{1}{4!} Z_\Phi^2 Z_{\lambda_s} \lambda_s \vec{\Phi}^4 - \frac{1}{2} Z_\Phi Z_H Z_\delta f_\delta \delta |H|^2 \vec{\Phi}^2 \end{aligned} \quad (3.13)$$

where  $\mathcal{L}_{\text{Bare}}^{\text{SM}}$  is the SM bare lagrangian, all couplings and fields are renormalized, and all constant factors  $Z_a$  have the form  $Z_a = 1 + z_a/\varepsilon + \dots$ , where the  $z_a$  factors are those of the  $\overline{\text{MS}}$  scheme. The introduction of the finite renormalization of the coupling  $\delta$ ,

$$f_\delta = \left( \frac{\mu}{\mu_\Phi} \right)^{4\delta/\kappa} \quad (3.14)$$

is necessary in particular to satisfy the RGE given by Eqs. (3.9).

The term in  $f_\delta$ , when expanded to one-loop order,  $f = 1 + (4\delta/\kappa) \log \frac{\mu}{\mu_\Phi} + \dots$  generates a (finite counterterm) contribution to the effective potential in the presence of background fields  $h_c$  and  $\Phi$  as,

$$\Delta_c V(h_c, \Phi) = \frac{1}{16\pi^2} \delta^2 h_c^2 \vec{\Phi}^2 \log \frac{\mu}{\mu_\Phi} \quad (3.15)$$

which grows logarithmically with the scales ratio  $\mu/\mu_\Phi$ . In principle, this is worrying, as it represents the kind of drawback of the pure  $\overline{\text{MS}}$ -scheme in the presence of several scales that we wanted to avoid with our approach. Besides, this seems to contradict the form of the effective potential (3.12), which is free of such dangerous logarithms. However, when turning the background field  $\Phi$  on and computing the one-loop diagram with external legs  $|H|^2 \vec{\Phi}^2$  and internal propagators corresponding to a Higgs and a  $\vec{\Phi}$  field<sup>5</sup>, the contribution of the latter (after renormalization in the  $\overline{\text{MS}}$ -scheme) precisely cancels that of the counterterm in Eq. (3.15) and, altogether, the presence of the dangerous  $\log(\mu/\mu_\Phi)$  term, leading to the one-loop effective potential given in Eq. (3.12).

## 4 Numerical results

In this section we will study the potential given by Eqs. (3.2, 3.12) and analyze the conditions for stability of the electroweak minimum at the vacuum expectation value (VEV)  $h_c = v = 246$  GeV, and the non-appearance of a (destabilizing) deeper minimum

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<sup>5</sup>In fact this is the only non-trivial one-loop diagram, in the sense that it contains internal lines corresponding to the SM and to new physics. This diagram is proportional to  $\delta^2$  and plays a major role in our construction.



mass squared  $m_H^2$ , the minimum conditions of the potential read<sup>6</sup>

$$\begin{aligned}\left.\frac{d V_{\text{eff}}}{d h_c}\right|_{h_c=v} &= 0 \\ \left.\frac{d^2 V_{\text{eff}}}{d h_c^2}\right|_{h_c=v} &= m_H^2\end{aligned}\quad (4.1)$$

Using these conditions, the effective potential parameters,  $m^2$  and  $\lambda$ , can be traded by  $v$  and  $m_H^2$  as:

$$\begin{aligned}m^2 &= m_{\text{SM}}^2 - \frac{N \delta^2}{\kappa} v^2 + \frac{N \delta}{\kappa} M^2 \left( \log \frac{M_\Phi^2}{\mu_\Phi^2} - 1 \right) \\ \lambda &= \lambda_{\text{SM}} - \frac{N \delta^2}{\kappa} \log \frac{M_\Phi^2}{\mu_\Phi^2}\end{aligned}\quad (4.2)$$

where  $m_{\text{SM}}^2$ ,  $\lambda_{\text{SM}}$  represent the corresponding values as obtained in the pure SM

$$\begin{aligned}m_{\text{SM}}^2 &= \frac{1}{2} m_H^2 + \frac{3 \lambda_t^4}{\kappa} v^2 \\ \lambda_{\text{SM}} &= \frac{m_H^2}{2 v^2} + \frac{3 \lambda_t^4}{\kappa} \log \frac{M_t^2}{\mu^2}.\end{aligned}\quad (4.3)$$

From the second equality in Eq. (4.2) we see that in order to preserve the validity of perturbation theory, a choice of the scale,  $\mu_\Phi \simeq M_\Phi$ , must be done, as expected. Furthermore, from the first equality in Eq. (4.2), we see that for  $M^2 \gg m_H^2$  (which is the usual case) the third term of the right hand side will amount to a huge contribution, which must be fine-tuned with the value of  $m^2$ , in order to keep the right scale for  $m_H$ . This technical problem, which reflects a hierarchy problem, is avoided by choosing  $\mu_\Phi$  in such a way that the annoying term in (4.2) is cancelled, i.e.

$$\log \frac{M_\Phi^2}{\mu_\Phi^2} = 1. \quad (4.4)$$

Then the parameter fixing (4.2) becomes,

$$\begin{aligned}m^2 &= m_{\text{SM}}^2 - \frac{N \delta^2}{\kappa} v^2 \\ \lambda &= \lambda_{\text{SM}} - \frac{N \delta^2}{\kappa}.\end{aligned}\quad (4.5)$$

Still perturbation theory can be spoiled, along with the SM vacuum, for values of  $N$  and  $\delta$  such that  $N \delta^2 \gtrsim \kappa$  so we will restrict ourselves to the range of values such that  $N \delta^2 \ll \kappa$ .

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<sup>6</sup>With the definition of Eq.(4.1) the physical (pole) squared Higgs mass  $M_H^2$  is equal to  $m_H^2$  plus some (small) radiative corrections, which have been taken into account in the numerical computations.

theory,  $\lambda_a$ , acquire an implicit  $h_c$ -dependence through their dependence on  $\mu_\Phi$ . In fact this dependence can be obtained from the second equalities of Eqs. (3.5–3.11) using

$$\frac{d\lambda_a(h_c)}{d\log h_c} = \left[ 1 - \frac{M^2}{M_\Phi^2(h_c)} \right]_{\mu_\Phi} \tilde{\beta}_{\lambda_a} \theta(\mu_\Phi - \mu) . \quad (4.6)$$

We will consider now the effective potential given by Eqs. (3.2) and (3.12) and will improve it by the RGE (3.5) to (3.11) and (4.6) with the choice (4.4) of the  $\mu_\Phi$  scale. The initial conditions for all parameters will be taken at the boundary scales:

$$\begin{aligned} \mu_0^2 &= M_t^2(v) \\ \mu_{\Phi,0}^2 &= M_\phi^2(v)/e \end{aligned} \quad (4.7)$$

and those for  $m^2$  and  $\lambda$  will be fixed by (4.5). The system of partial differential equations (3.5) to (3.11) is solved by a step-wise procedure, which allows to evaluate the effective potential for any value of  $h_c$ .

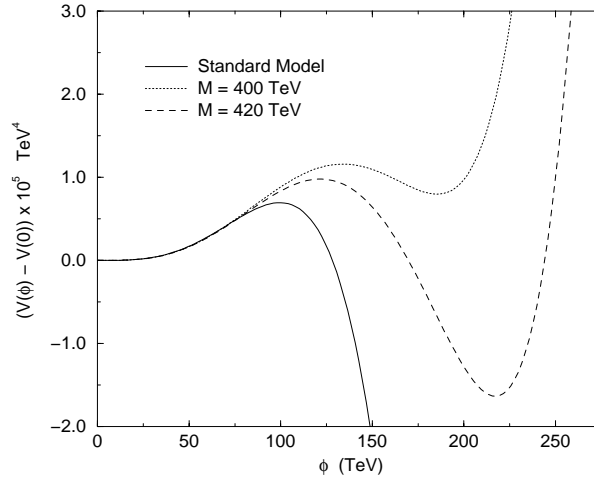


Figure 1: Plot of the effective potential as function of the Higgs field for  $m_H = 100$  GeV,  $\delta = 1$  and  $N = 10$ .

In Fig. 1 we show a plot of the effective potential  $V_{\text{eff}}$  for values of the Higgs and top-quark masses,  $m_H = 100$  GeV and  $M_t(v) = 175$  GeV, for the SM (solid line), which shows an instability for values of the field  $h_c \simeq 125$  TeV  $\equiv \Lambda_{SM}$ . The dotted line corresponds to  $V_{\text{eff}}$  for the SM extension with  $\delta = 1$ ,  $N = 10$  and  $M = 400$  TeV, which shows how the instability is cured by the new physics. Smaller values of  $M$  also work. Conversely, the SM results are recovered in the limit when  $M \rightarrow \infty$  or  $\delta \rightarrow 0$ . This is illustrated by the dashed line, which corresponds to  $M = 420$  TeV.

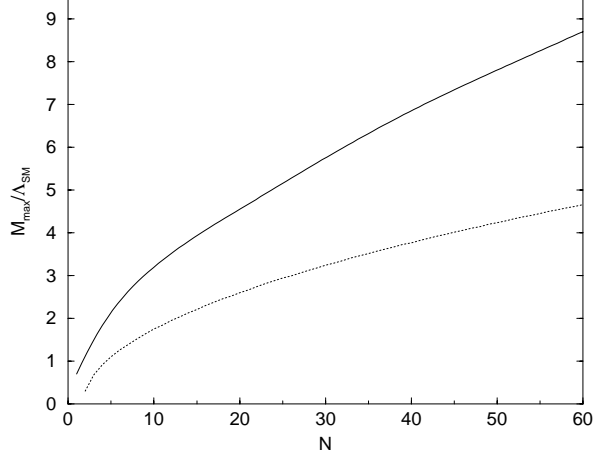


Figure 2: Plot of the ratio  $M_{max}/\Lambda_{SM}$  for  $\delta = 1$  and  $m_H = 100$  GeV as function of the multiplicity  $N$ . The solid line and the dashed one shows the RGE improved and one-loop approximation respectively.

The previous example explicitly shows that the scale of new physics,  $M$ , responsible for the cure of a SM instability, can be larger than the scale  $\Lambda_{SM}$  at which the SM instability develops. It cannot be, however, arbitrarily larger. Fig. 2 shows (solid line) the ratio  $M_{max}/\Lambda_{SM}$  for  $\delta = 1$ , as a function of the number of extra degrees of freedom,  $N$  (recall that  $N\delta^2 \ll \kappa$  to preserve perturbativity). Clearly,  $M$  could be as large as  $\simeq 10\Lambda_{SM}$ , which puts an upper bound on the scale of new physics,  $M$ . For a typical value of the multiplicity,  $N = \mathcal{O}(10)$ , we get the conservative bound  $M \lesssim 4\Lambda_{SM}$ . This is e.g. the case of the MSSM, where the relevant multiplicity is  $N = 12$ , corresponding to the stops. The dashed line corresponds to the result of Ref. [3], obtained in a cruder approximation (one-loop instead of RGE improved). Our results confirm the trend observed in that paper, but show that the ratio  $M_{max}/\Lambda_{SM}$  can be substantially larger than the one estimated there.

Finally, Fig. 3 shows the smooth increasing of both,  $\Lambda_{SM}$  and  $M_{max}$ , with the Higgs mass,  $M_H$ . Also, there appears a lower bound on  $M$ , coming from the requirement of perturbativity up to the  $\Lambda_{SM}$  scale. This lower bound may seem paradoxical. Actually, it is a feature of the particular SM extension we have chosen: the lower  $M$ , the sooner the new physics enters, which, due to the RGE (3.5), may drive more quickly the quartic Higgs coupling  $\lambda$  into a non-perturbative regime. Other SM extensions, in particular supersymmetric extensions, do not present such lower limits on  $M$ . On the other hand, the upper bound on  $M$  is quite robust for any conceivable SM extension, as has been explained at the beginning of section 3.

The numerical results presented in Figs. 1–3 show the relation between the scale

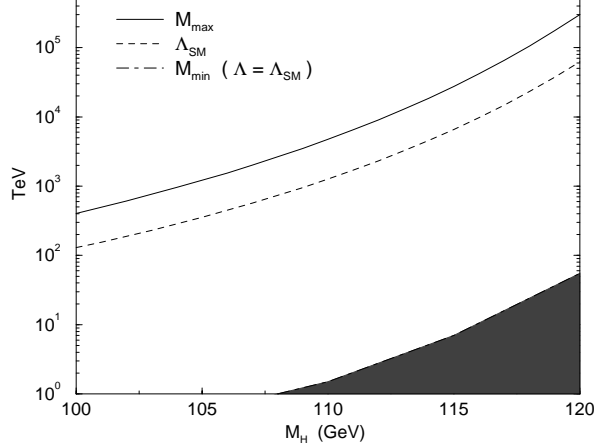


Figure 3: Plot of  $M_{max}$ ,  $\Lambda_{SM}$  and  $M_{min}$  as a function of the Higgs mass. The shadowed region shows the values not allowed for  $M$ .

$\Lambda_{SM}$ , at which the SM develops a instability, and the maximum value allowed for the scale of new physics required to cure it. With the present experimental lower bounds on  $M_H$ ,  $M_H \gtrsim 105$  GeV, it is clear that the possible new physics could easily escape detection in the present and future accelerators.

## 5 Conclusions

The possible detection of a relatively light Higgs ( $M_H \lesssim 140$  GeV) would imply an instability of the Higgs effective potential, thus signaling the existence of new physics able to cure it. It is, therefore, a relevant question what is the relation between the Higgs mass (or, equivalently, the scale at which the instability develops,  $\Lambda_{SM}$ ) and the maximum allowed value of the scale of the new physics,  $M_{max}$ .

In this paper we have examined this question in a rigorous way. This requires, in the first place, a reliable approach to evaluate the effective potential when several different mass scales are present. We have followed the decoupling approach exposed in a previous paper [7], which can also be re-formulated as a multi-scale renormalization approach, similar to those of Refs. [9, 10]. Then, we have considered a simple extension of the Standard Model, consisting of  $N$  extra scalar fields with an invariant mass  $M$ , coupled to the Higgs field with a coupling  $\delta$ . This model, although unrealistic, arguably represents the most efficient way to cure a SM instability, thus giving reliable upper bounds on the scale of the required new physics, i.e. the mass of the extra particles.

The numerical results, presented in section 4, in particular in Figs. 1–3, show the relation between  $\Lambda_{SM}$  and  $M_{max}$ . More precisely, for  $\delta = \mathcal{O}(1)$  and  $N = \mathcal{O}(10)$  (similar

upper bound on the scale of new physics. Unfortunately, the present lower bounds on the Higgs mass,  $M_H \gtrsim 105$  GeV, imply that  $\Lambda_{SM}$  is at least  $10^2$  TeV. This fact, together with the previous upper bound on  $M$ , implies that the new physics could easily (but not necessarily) escape detection in the present and future accelerators.

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